Dynamic Structural Transformations for Distillation Control Configurations

A control structure transformation is presented that includes the effects of inventory control system dynamics and internal flow dynamics on flows and compositions. This new transformation can be used to predict column dynamic operation for any alternative control structure from the open-loop model (inventory control only) obtained for a particular control structure (LV). New types of inverse responses are predicted using this technique and verified both by rigorous dynamic simulation and by experimental studies on a pilot-scale distillation column. These features also can be shown theoretically using a simplified transfer function model.

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Introduction

Distillation is one of the most active research areas in process control and deservedly so. According to a survey among plant managers in the process industries, distillation is the process that would benefit most from improved control (Dartt, 1985). Control of a distillation column consists of two main objectives: inventory control (level control of reflux drum and bottom sump) and product composition control. Different control structures, in which alternative arrangements of inventory and composition controllers are used, can yield very different operating characteristics—for example, in multiloop SISO control, the LV structure (where reflux rate L and vapor boilup V are used for top and bottom composition control, respectively) or the DV structure (where distillate rate D and boilup are used for top and bottom composition control, respectively). Choice of the control structure is a major concern and considerable research activity has been devoted to finding the best control structure for specified control objectives. Thus it is important to be able to evaluate alternatives easily in attempting to identify the best control structure.

The vast literature on distillation control has been reviewed several times during the past decade. Tolliver and Waggoner (1980) summarize research for distillation column control during the period 1970–79; Waller (1982) reviewed the literature on dual-composition control up to 1981; and McAvoy and Wang (1986) reviewed publications during 1980–84. Distillation control structures were the subject of a 1986 review (Waller, 1986). Among these, a wide variety of distillation control structures have been reported in the literature. In addition to using simple flows as composition manipulators, as in the LV and DV structures, ratios of flows often have been suggested. An early example is Rijnsdorp's suggestion (1965) to use the ratio of reflux flow and overhead vapor flow $L/(L + D)$ as a manipulator for the top composition control. The suggestion was studied experimentally by Wood and Berry (1973) who compared the $[(L + D)/L]V$ structure with the conventional LV structure. The book by Rademaker et al. (1975) lists a large number of flow ratio manipulators discussed in the literature. Renewed interest in ratio control returned in the 1980's. Ryskamp (1980) proposed a dual composition control scheme which uses the ratio of the distillate flow to the overhead vapor flow $D/(L + D)$ to control distillate composition and the vapor boilup to control bottom composition. Waller et al. (1988) experimentally compared four structures—LV, DV, $[D/(L + D)]V$ and $[D/(L + D)](V/B)$—on a pilot-plant column. The choice of manipulators also has been investigated recently by Skogestad and Morari (1987), Häggblom and Waller (1989), and Skogestad et al. (1989). An unusual choice of manipulators was recently proposed by Finco et al. (1989) who investigated the DB structure which previously had been rejected by researchers in the field as representing an infeasible structure. They reported that this structure has favorable characteristics for the control of low relative volatility distillation columns if override controls are used in the composition control loops to reduce the fragility of the control structure. In addition to choosing combinations of flows as new manipulators, a number of suggestions have been made to combine the

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controlled variables in various other ways. One example is to control sums and differences of measured variables such as compositions or tray temperatures. A review and illustration of these choices is given by Waller and Finnerman (1987).

Recently, Häggblom and Waller (1986, 1988a,b) and Häggblom (1988) have suggested a structural transformation method that is based on a knowledge of steady-state gains and consistency relationships. This method is very useful for studying alternative control structures for continuous distillation because, once a steady-state model is available for one arbitrary structure, process models for any other structures can be readily calculated. The transformation was developed for the steady-state part of the process model, i.e., for the process gains; but, it is also valid for the dynamic case: 1. when the secondary output variables (the column inventories) are assumed to be perfectly controlled or 2. when the manipulators for the secondary variables are not changed by the transformation. Using these transformations, a control structure was proposed which is decoupled and rejects disturbances in the steady state (Häggblom and Waller, 1990). This control method represents a type of inferential feedforward control where disturbances are inferred from knowledge of product flow rates, D and B.

Two kinds of dynamics are important in distillation column behavior: fast dynamics that describe column internal flows and slow dynamics associated with composition changes. Skogestad and Morari (1988) have studied column composition dynamics assuming a combination of both fast and slow dynamic effects. In many cases, the fast dynamics can be neglected. Alternatively, the column behavior can be described with two separate dynamic effects where the fast dynamic quantities (associated with flows) and the slow dynamic properties (associated with compositions) affect each other. The effects of the slow dynamics on the fast dynamics, however, can be neglected, such as flow changes caused by variations in heat transfer effects which are in turn related to composition changes. Using this approach, a control structure that is different from a base structure (such as the LV structure) can be derived that accommodates the important inventory and flow dynamics.

In this paper, we extend the previous work of Häggblom and Waller by developing a method that incorporates process dynamics in both the structural transformation and the disturbance rejection and decoupling structure, but that does not assume either perfect inventory control or fixed choice of inventory manipulators. A new derivation provides a physical interpretation of the gains in the equation for the manipulated variables that are used for inventory control. Also, gain consistency relationships are derived for multicomponent systems where arbitrary components are selected for the controlled compositions and feed disturbances. This research demonstrates for the first time the importance of the inventory controls for various control configurations.

The new transformation has been used to predict that inverse responses can occur for open-loop conditions (i.e., with inventory control only) in several control structures where this important feature has not been noted before. These predicted inverse responses have been verified in both simulation and experimental studies.

Structural Transformations

To develop a dynamic structural transformation technique, a concise analysis of the distillation control structure is needed; for example, the common and distinctive elements among different control structures should be identified. The derived model should be able to predict dynamic characteristics, and at the same time the parameters of the model should be readily obtainable. In this section a base model and its parameters will be defined and then the dynamic structural transformation technique will be presented.

Base structure for distillation columns

A schematic diagram of a distillation column is shown in Figure 1 where the dashed envelope contains the invariant structural element, which is independent of control structure. In this and subsequent discussion the column pressure is assumed to be constant. Momentarily ignoring inventory controls for the column, the input variables for the invariant element are reflex rate L, boilup V, feed rate F, and feed composition z; the output variables are top product composition y and bottom product composition x. F and z are considered to be disturbances. Different components can be chosen to represent the composition at each end of the column and the feed composition disturbances (single component control, impurity control, etc.). It is well known that the relations between the inputs (manipulated variables) and outputs (controlled variables) are nonlinear and higher order, but simple transfer function models such as first-order plus time-delay models can be useful for purposes of analysis. A number of methods, such as the process reaction curve method, can be used to estimate the model parameters (Seborg et al., 1989).

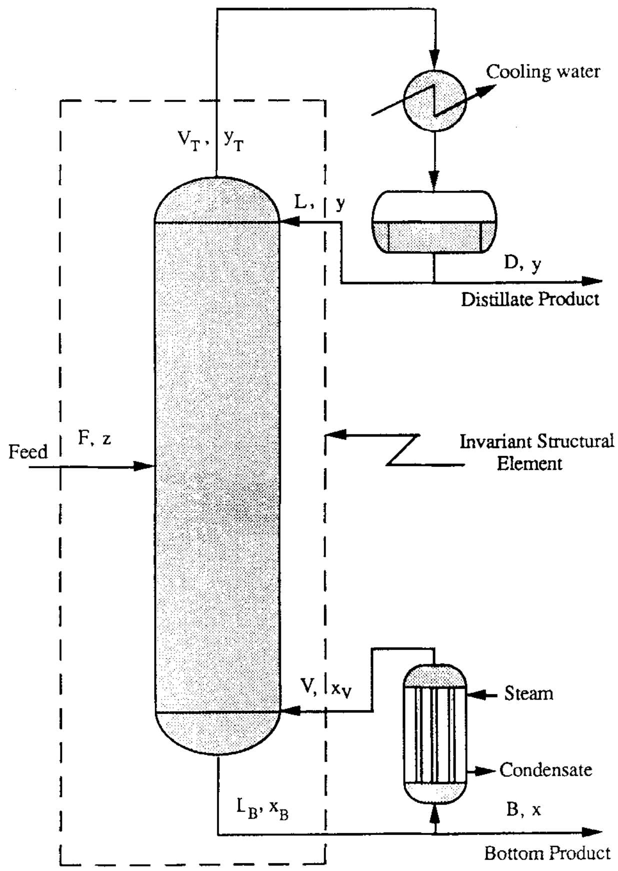

Figure 1. Distillation column.

The transfer function model for the invariant structural element can be expressed as

$$\begin{bmatrix} \Delta y_T \\ \Delta x_B \end{bmatrix} = \begin{bmatrix} G^o_{yL} & G^o_{yV} \\ G^o_{xL} & G^o_{xV} \end{bmatrix} \begin{bmatrix} \Delta L \\ \Delta V \end{bmatrix} + \begin{bmatrix} G^o_{yF} & G^o_{yz} \\ G^o_{xF} & G^o_{xz} \end{bmatrix} \begin{bmatrix} \Delta F \\ \Delta z \end{bmatrix} \tag{1}$$

where Δy_T is the vapor composition leaving the top tray, Δx_B is the liquid composition leaving the bottom tray, the G's represent transfer functions with the indicated output/input subscripts, Δ denotes a deviation variable based on nominal steady-state values, and superscript o indicates the invariant structure. The actual product compositions, Δy and Δx, differ from Δy_T and Δx_B due to the mixing associated with the accumulators (reflux drum plus condenser at the top and reboiler plus sump at the bottom)

$$\begin{aligned} \Delta y &= G_{Dy} \Delta y_T \\ \Delta x &= G_{Bx} \Delta x_B \end{aligned} \tag{2}$$

where G_{Dy} and G_{Bx} are the transfer functions for the composition dynamics in the reflux drum plus condenser and reboiler plus sump, respectively. However, if changes in the residence times of the accumulators are relatively small, then the mixing dynamics can be assumed to be independent of control structure. Thus by combining Eqs. 1 and 2, another invariant element is obtained

$$\begin{bmatrix} \Delta y \\ \Delta x \end{bmatrix} = \begin{bmatrix} G_{yL} & G_{yV} \\ G_{xL} & G_{xV} \end{bmatrix} \begin{bmatrix} \Delta L \\ \Delta V \end{bmatrix} + \begin{bmatrix} G_{yF} & G_{yz} \\ G_{xF} & G_{xz} \end{bmatrix} \begin{bmatrix} \Delta F \\ \Delta z \end{bmatrix} \tag{3}$$

where

$$\begin{aligned} G_{yL} &= G_{Dy} G^o_{yL} \quad G_{yV} = G_{Dy} G^o_{yV} \\ G_{xL} &= G_{Bx} G^o_{xL} \quad G_{xV} = G_{Bx} G^o_{xV} \end{aligned} \tag{4}$$

A rigorous model of the invariant element could have quite complicated relations between the flows (L, V, V_T, L_B) and compositions (y, x, y_{V_T}, x_{L_B}, x_V). However, the manipulated inputs are L and V; the outputs of interest are y and x; and the other variables are related to one another in a very complicated way. Thus after elimination of variables of no interest, the input/output model of the invariant element is obtained as Eq. 3 which describes the composition dynamics of the LV structure and can serve as a base model for structural transformation. If some other structure (e.g., DV) is used as a base model, the transfer functions of the composition dynamics contain the inventory dynamics and it is impossible to achieve proper structural transformations.

In previous work by Häggblom and Waller (1988a,b), perfect inventory control was assumed and the internal flow dynamics of the column were neglected. The internal flow dynamics often can be legitimately neglected; however, in considering different control structures, the dynamic effects of inventory control can play a very important role. For example, with the LV structure the effects of inventory controls on the outputs (compositions) are negligible; but with the DV structure, the inventory control dynamics caused by the reflux drum level controller significantly affect the reflux flow rate in attempting to maintain the level (as is demonstrated later in this paper).

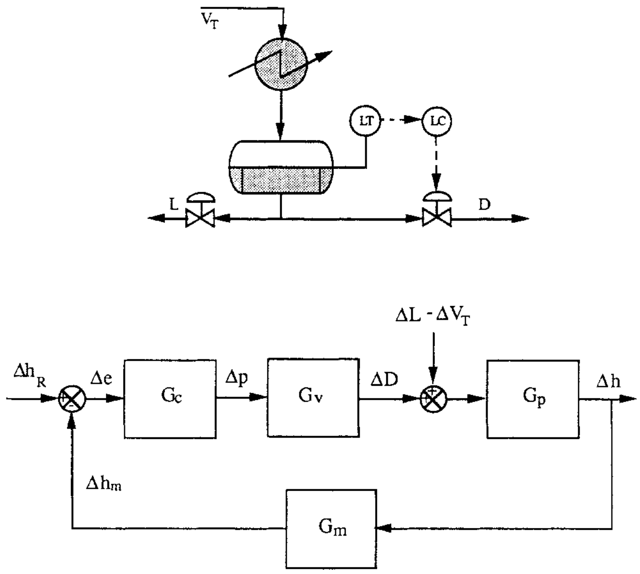

Figure 2. Drum level control system for the LV structure.

Figure 2 shows schematic and block diagrams for the reflux drum level control system of the LV structure. For the LV structure, the inventory control equations for the reflux drum and reboiler can be written in vector form

$$\begin{bmatrix} \Delta D \\ \Delta B \end{bmatrix} = \begin{bmatrix} G_D(\Delta V_T - \Delta L) \\ G_B(\Delta L_B - \Delta V) \end{bmatrix} \tag{5}$$

where D and B denote distillate and bottom flow rates, respectively, V_T is the vapor flow rate leaving the top tray, and L_B is the liquid flow rate leaving the bottom tray. G_D and G_B are transfer functions with steady-state gains of one representing the closed-loop dynamics of the inventory control systems for the reflux drum and column base, respectively. These can be obtained using block diagram algebra for the liquid level control system in Figure 2. (The derivation of G_D and G_B is presented in appendix A.) If constant molar overflow is assumed, then $\Delta V_T = \Delta V$ and $\Delta L_B = \Delta L$; however, this assumption is unrealistic for most actual columns (Häggblom and Waller, 1988a,b), even though it can be extended somewhat by introducing fictitious molecular weights (Robinson and Gilliland, 1950). The internal flow dynamics for the invariant structural element, i.e., the relationships among ΔV_T, ΔL_B, ΔL, ΔV, ΔF and Δz, can be modeled similar to Eq. 3:

$$\begin{bmatrix} \Delta V_T \\ \Delta L_B \end{bmatrix} = \begin{bmatrix} G_{V_T L} & G_{V_T V} \\ G_{L_B L} & G_{L_B V} \end{bmatrix} \begin{bmatrix} \Delta L \\ \Delta V \end{bmatrix} + \begin{bmatrix} G_{V_T F} & G_{V_T z} \\ G_{L_B F} & G_{L_B z} \end{bmatrix} \begin{bmatrix} \Delta F \\ \Delta z \end{bmatrix} \tag{6}$$

The transfer functions in Eq. 6 can be determined from information on column holdups and internal flow rates or obtained experimentally. The steady-state gains of these transfer functions can be calculated from steady-state information alone, as will be shown. Since the dynamics in Eq. 6 typically are fast compared to the composition dynamics, they often can be neglected. With this assumption for simplicity, we obtain the following relationships,

$$\Delta V_T = K_{V_TL}\Delta L + K_{V_TV}\Delta V + K_{V_TF}\Delta F + K_{V_Tz}\Delta z \quad (7)$$

$$\Delta L_B = K_{L_BL}\Delta L + K_{L_BV}\Delta V + K_{L_BF}\Delta F + K_{L_Bz}\Delta z \quad (8)$$

where the K's represent the gains corresponding to the indicated subscripts. The steady-state material balance around the invariant structural element in Figure 1 is

$$\Delta L_B + \Delta V_T = \Delta L + \Delta V + \Delta F \quad (9)$$

Thus the gains in Eqs. 7 and 8 are related by Eq. 9 as follows:

$$\begin{aligned} K_{L_BL} &= 1 - K_{V_TL} \quad K_{L_BV} = 1 - K_{V_TV} \\ K_{L_BF} &= 1 - K_{V_TF} \quad K_{L_Bz} = -K_{V_Tz} \end{aligned} \quad (10)$$

For cases where the dynamics in Eq. 6 cannot be ignored, possible simplification is discussed in a later section.

Combining Eqs. 5, 7 and 8, we obtain the relations among the external flows (ΔD and ΔB), the internal flows (ΔL and ΔV), and the disturbances (ΔF and Δz),

$$\begin{bmatrix} \Delta D \\ \Delta B \end{bmatrix} = \begin{bmatrix} G_{DL} & G_{DV} \\ G_{BL} & G_{BV} \end{bmatrix} \begin{bmatrix} \Delta L \\ \Delta V \end{bmatrix} + \begin{bmatrix} G_{DF} & G_{Dz} \\ G_{BF} & G_{Bz} \end{bmatrix} \begin{bmatrix} \Delta F \\ \Delta z \end{bmatrix} \quad (11)$$

where

$$\begin{aligned} G_{DL} &= G_D(K_{V_TL} - 1) & G_{DV} &= G_D K_{V_TV} \\ G_{BL} &= G_B(1 - K_{V_TL}) & G_{BV} &= -G_B K_{V_TV} \\ G_{DF} &= G_D K_{V_TF} & G_{Dz} &= G_D K_{V_Tz} \\ G_{BF} &= G_B(1 - K_{V_TF}) & G_{Bz} &= -G_B K_{V_Tz} \end{aligned} \quad (12)$$

Equations 3 and 11 represent a new dynamic model for an LV control structure with inventory control. In contrast, the original Häggblom and Waller model utilized only steady-state gains.

The values of the gains in the transfer functions of Eq. 11 can be calculated from the steady-state relationships of any multicomponent system. From the overall and component balances, we have the following relations,

$$F = D + B \quad (13)$$

$$Fz_i = Dy_i + Bx_i \quad (i = 1, \ldots, n) \quad (14)$$

$$\Delta F = \Delta D + \Delta B \quad (15)$$

$$z_i\Delta F + F\Delta z_i = y_i\Delta D + D\Delta y_i + B\Delta x_i + x_i\Delta B \quad (i = 1, \ldots, n) \quad (16)$$

where n is the number of components. Equation 16 is an approximation obtained by a first-order Taylor series expansion of Eq. 14 around nominal conditions. From Eqs. 3, 11, 12, 13, 15 and 16 we obtain the following gain consistency relations,

$$K_{V_TL} = 1 - \left(\frac{\overline{D}K_{y_iL} + \overline{B}K_{x_iL}}{\overline{y}_i - \overline{x}_i}\right) \quad (i = 1, \ldots, n) \quad (17)$$

$$K_{V_TV} = -\left(\frac{\overline{D}K_{y_iV} + \overline{B}K_{x_iV}}{\overline{y}_i - \overline{x}_i}\right) \quad (i = 1, \ldots, n) \quad (18)$$

$$K_{V_TF} = \left(\frac{\overline{z}_i - \overline{x}_i - \overline{D}K_{y_iF} - \overline{B}K_{x_iF}}{\overline{y}_i - \overline{x}_i}\right) \quad (19)$$

$$K_{V_Tz} = \left(\frac{\overline{F}\left(\frac{\Delta z_i}{\Delta z_k}\right) - \overline{D}K_{y_iz} - \overline{B}K_{x_iz}}{\overline{y}_i - \overline{x}_i}\right) \quad (i = 1, \ldots, n) \quad (20)$$

where subscript k denotes the designated disturbance component in the feed, and an overbar denotes a steady-state value. The detailed derivation of Eqs. 17–20 is given in appendix B.

For the special case of constant molar overflow with compositions expressed as mole fractions and the reflux exactly at the boiling point, $K_{V_TV} = 1$ and $K_{V_TL} = 0$. Note that, if compositions are expressed as mass fractions, these values of the gains may not be applicable. In general, for an actual column, $K_{V_TV} \neq 1$ and $K_{V_TL} \neq 0$. The value of K_{V_TL} is a strong function of the degree of subcooling of the reflux. For example, the value of K_{V_TL} is decreased when the degree of subcooling is increased. Also K_{V_TF} is a function of feed quality. Typical values of K_{V_TL} and K_{V_TV} from the literature were compiled by Häggblom and Waller (1988b) and are shown in Table 1.

Equations 17–20 are valid for multicomponent systems with an arbitrary choice of components for the controlled compositions and feed disturbance. For binary systems, relations similar to Eqs. 17–20 were derived by Häggblom and Waller (1988b). The gains of the two components in the top and bottom composition have the same numerical value but opposite sign. However, if a multicomponent system is considered, the gain for one component cannot be inferred from that of another component. For example, if we choose as controlled variables different components for the top and bottom compositions (i for top and j for bottom), then K_{x_iL}, K_{x_iV}, K_{y_iF}, and K_{y_iz} are not explicitly available from the input-output model. This characteristic implies that additional information about the gains is needed, beyond that contained in input-output model relations, for the multicomponent system. In Eq. 20 the ratio of $(\Delta z_i/\Delta z_k)$ can be calculated easily. For example, if component k is changed by Δz_k and the ratios $(\Delta z_i/\Delta z_j)$ for $i, j \neq k$, are kept constant, then

$$\begin{aligned} \Delta z_i/\Delta z_k &= -\overline{z}_i/(1 - \overline{z}_k) \quad &(\text{for } i \neq k) \\ \Delta z_i/\Delta z_k &= 1 \quad &(\text{for } i = k) \end{aligned} \quad (21)$$

Table 1. Values of K_{V_TL} and K_{V_TV} from the Literature

Model	K_{V_TL}	K_{V_TV}
Wood and Berry*	0.75	0.49
Åbo Akademi*	0.39	1.35
McAvoy, Column A*	0.19	0.81
Column B*	0.29	0.71
Column C*	0.07	1.02
UCSB Column	0.23	0.82

*Values calculated by Häggblom and Waller (1988c).

A typical transformation: from LV to DV structure

In the DV structure, the liquid inventories are controlled by L and B (instead of D and B, as in the LV structure). Equation 5 is now modified by replacing the first row with the following relation for inventory control,

$$\Delta L = G_D(\Delta V_T - \Delta D) \quad (22)$$

The closed-loop transfer function, G_D, need not be the same as for the LV case. If for example, different level controller settings are used, then those settings will be reflected in G_D. The elimination of ΔV_T and ΔL_B from the modified Eq. 5 using Eqs. 7 and 8; and Eq. 10 gives

$$\begin{bmatrix}\Delta L\\ \Delta B\end{bmatrix} = \begin{bmatrix}G_{LD}^{DV} & G_{LV}^{DV}\\ G_{BD}^{DV} & G_{BV}^{DV}\end{bmatrix}\begin{bmatrix}\Delta D\\ \Delta V\end{bmatrix} + \begin{bmatrix}G_{LF}^{DV} & G_{Lz}^{DV}\\ G_{BF}^{DV} & G_{Bz}^{DV}\end{bmatrix}\begin{bmatrix}\Delta F\\ \Delta z\end{bmatrix} \quad (23)$$

where superscript DV denotes the DV structure and

$$G_{LD}^{DV} = -\frac{G_D}{1 - K_{V_TL}G_D} \qquad G_{LV}^{DV} = \frac{G_D K_{V_TV}}{1 - K_{V_TL}G_D}$$
$$G_{LF}^{DV} = \frac{G_D K_{V_TF}}{1 - K_{V_TL}G_D} \qquad G_{Lz}^{DV} = \frac{G_D K_{V_Tz}}{1 - K_{V_TL}G_D}$$
$$G_{BD}^{DV} = -\frac{G_B G_D(1 - K_{V_TL})}{1 - K_{V_TL}G_D} \qquad G_{BV}^{DV} = -\frac{G_B(1 - G_D)K_{V_TV}}{1 - K_{V_TL}G_D}$$
$$G_{BF}^{DV} = \frac{G_B\{1 - (1 - G_D)K_{V_TF}\}}{1 - K_{V_TL}G_D} \qquad G_{Bz}^{DV} = -\frac{G_B(1 - G_D)K_{V_Tz}}{1 - K_{V_TL}G_D} \quad (24)$$

Further, ΔL in Eq. 3 can be eliminated by using Eq. 23 to obtain the relationship between inputs and outputs of the DV structure,

$$\begin{bmatrix}\Delta y\\ \Delta x\end{bmatrix} = \begin{bmatrix}G_{yD}^{DV} & G_{yV}^{DV}\\ G_{xD}^{DV} & G_{xV}^{DV}\end{bmatrix}\begin{bmatrix}\Delta D\\ \Delta V\end{bmatrix} + \begin{bmatrix}G_{yF}^{DV} & G_{yz}^{DV}\\ G_{xF}^{DV} & G_{xz}^{DV}\end{bmatrix}\begin{bmatrix}\Delta F\\ \Delta z\end{bmatrix} \quad (25)$$

where

$$G_{yD}^{DV} = -\frac{G_D}{1 - K_{V_TL}G_D}G_{yL} \qquad G_{yV}^{DV} = G_{yV} + \frac{G_D K_{V_TV}}{1 - K_{V_TL}G_D}G_{yL}$$
$$G_{xD}^{DV} = -\frac{G_D}{1 - K_{V_TL}G_D}G_{xL} \qquad G_{xV}^{DV} = G_{xV} + \frac{G_D K_{V_TV}}{1 - K_{V_TL}G_D}G_{xL}$$
$$G_{yF}^{DV} = G_{yF} + \frac{G_D K_{V_TF}}{1 - K_{V_TL}G_D}G_{yL} \qquad G_{yz}^{DV} = G_{yz} + \frac{G_D K_{V_Tz}}{1 - K_{V_TL}G_D}G_{yL}$$
$$G_{xF}^{DV} = G_{xF} + \frac{G_D K_{V_TF}}{1 - K_{V_TL}G_D}G_{xL} \qquad G_{xz}^{DV} = G_{xz} + \frac{G_D K_{V_Tz}}{1 - K_{V_TL}G_D}G_{xL} \quad (26)$$

Structural transformation to an arbitrary control structure

Häggblom and Waller (1988b) developed a structural transformation technique using steady-state gains. In this section we show that a transformation from one control structure to another also can be derived for dynamic models, if relations between the manipulated variables for the base structure and the new structure are available. In vector-matrix notation, Eqs. 3, 5 and 8 can be combined and written together as follows,

$$\begin{bmatrix}\Delta \mathbf{Y}\\ \Delta \mathbf{I}\end{bmatrix} = \begin{bmatrix}\mathbf{G}_{YU}\\ \mathbf{G}_{IU}\end{bmatrix}\Delta \mathbf{U} + \begin{bmatrix}\mathbf{G}_{YW}\\ \mathbf{G}_{IW}\end{bmatrix}\Delta \mathbf{W} \quad (27)$$

where $\Delta \mathbf{Y} = [\Delta y\ \Delta x]^T$ is a vector of controlled variables, $\Delta \mathbf{I} = [\Delta D\ \Delta B]^T$ is a vector of manipulated variables for inventory control, $\Delta \mathbf{U} = [\Delta L\ \Delta V]^T$ is a vector of manipulated variables for composition control, and $\Delta \mathbf{W} = [\Delta F\ \Delta z]^T$ is a vector of disturbances. Equation 27 represents the "base structure," in this case the LV structure, so that

$$\mathbf{G}_{YU} = \begin{bmatrix}G_{yL} & G_{yV}\\ G_{xL} & G_{xV}\end{bmatrix} \quad \mathbf{G}_{YW} = \begin{bmatrix}G_{yF} & G_{yz}\\ G_{xF} & G_{xz}\end{bmatrix}$$
$$\mathbf{G}_{IU} = \begin{bmatrix}G_{DL} & G_{DV}\\ G_{BL} & G_{BV}\end{bmatrix} \quad \mathbf{G}_{IW} = \begin{bmatrix}G_{DF} & G_{Dz}\\ G_{BF} & G_{Bz}\end{bmatrix} \quad (28)$$

where superscript T denotes the transpose of a vector.

For a steady-state structural transformation, the steady-state form of Eq. 5, or its equivalent, does not have to be changed for different control structures. However, if we want to change to another control structure and include inventory control dynamics, then Eq. 5 for inventory control should be modified to reflect the new configuration. For example, if the new control structure is the DV structure, a new expression for ΔD can be obtained from Eqs. 7 and 22:

$$\Delta D = \left(K_{V_TL} - \frac{1}{G_D}\right)\Delta L + K_{V_TV}\Delta V + K_{V_TF}\Delta F + K_{V_Tz}\Delta z \quad (29)$$

Using this equation, the first row of $\mathbf{G}_{IU}$ and $\mathbf{G}_{IW}$ in Eq. 27 should be modified before transformation to the DV structure. Then, in analogy with Häggblom and Waller's transformation, the transformed DV structure can be obtained. Similarly, if the new structure is LB, the new expression for ΔB can be derived:

$$\Delta B = (1 - K_{V_TL})\Delta L + \left(1 - K_{V_TV} - \frac{1}{G_B}\right)\Delta V + (1 - K_{V_TF})\Delta F - K_{V_Tz}\Delta z \quad (30)$$

and substituted for the second row of $\mathbf{G}_{IU}$ and $\mathbf{G}_{IW}$.

In the general case (new manipulated or disturbance variables), the relationships between the new and old variables can be expressed by

$$\begin{bmatrix}\Delta \boldsymbol{\nu}\\ \Delta \boldsymbol{\mu}\\ \Delta \boldsymbol{\omega}\end{bmatrix} = \begin{bmatrix}\mathbf{H}_{\nu I} & \mathbf{H}_{\nu U} & \mathbf{H}_{\nu W}\\ \mathbf{H}_{\mu I} & \mathbf{H}_{\mu U} & \mathbf{H}_{\mu W}\\ \mathbf{0} & \mathbf{0} & \mathbf{H}_{\omega W}\end{bmatrix}\begin{bmatrix}\Delta \mathbf{I}\\ \Delta \mathbf{U}\\ \Delta \mathbf{W}\end{bmatrix} \quad (31)$$

where the $\mathbf{H}$ matrices in Eq. 31 denote the algebraic relations (gains) between the new and old variables, $\Delta \boldsymbol{\nu} = [\Delta\nu_1\ \Delta\nu_2]^T$ is the vector of manipulated variables for inventory control of the new structure, $\Delta \boldsymbol{\mu} = [\Delta\mu_1\ \Delta\mu_2]^T$ is the vector of manipulated variables for composition control of the new structure, and $\Delta \boldsymbol{\omega} = [\Delta\omega_1\ \Delta\omega_2]^T$ is the vector of disturbance variables of the new structure. The output equation for the new control structure is expressed as follows,

$$\begin{bmatrix}\Delta Y\\ \Delta \nu\end{bmatrix} = \begin{bmatrix}G_{Y\mu}\\ G_{\nu\mu}\end{bmatrix}\Delta\mu + \begin{bmatrix}G_{Y\omega}\\ G_{\nu\omega}\end{bmatrix}\Delta\omega \tag{32}$$

From Eqs. 27, 31 and 32 we can determine the matrices in Eq. 32 as follows,

$$\begin{bmatrix}G_{Y\mu}\\ G_{\nu\mu}\end{bmatrix} = \begin{bmatrix}G_{YU}\\ H_{\nu U} + H_{\nu I}G^*_{IU}\end{bmatrix}(H_{\mu U} + H_{\mu I}G_{IU})^{-1} \tag{33}$$

$$\begin{bmatrix}G_{Y\omega}\\ G_{\nu\omega}\end{bmatrix} = \left\{\begin{bmatrix}G_{YW}\\ H_{\nu W} + H_{\nu I}G^*_{IW}\end{bmatrix} - \begin{bmatrix}G_{Y\mu}\\ G_{\nu\mu}\end{bmatrix}(H_{\mu W} + H_{\mu I}G_{IW})\right\}H_{\omega W}^{-1} \tag{34}$$

where the superscript * indicates that a transfer function has been modified to correspond to the new inventory control structure.

For the case where a nonlinear transformation is desired, e.g., a ratio of flow rates is used for one or more of the manipulated variables, the gain matrices in Eq. 31 represent partial derivatives evaluated at the nominal operating condition as shown by Häggblom (1988).

Häggblom and Waller (1988b) also developed a disturbance rejection and decoupling (DRD) structure using steady-state relations. From the above transformation we can, in principle, develop a dynamic DRD structure by setting $G_{Y_\mu} = I$ (no interaction) and $G_{Y_\omega} = 0$ (no disturbance propagation), and solving for $H_{\mu I}$ and $H_{\mu U}$. Theoretically, the dynamic DRD structure can reject unmeasured disturbances in F and z perfectly without showing any interactions between the two composition loops. In practice, modeling errors will degrade the performance somewhat.

Prediction of Inverse Response for Certain Control Structures

For the DV structure, if G_{yV}^{DV} in Eq. 26 is considered as an example,

$$G_{yV}^{DV} = G_{yV} + \frac{G_D K_{V_TV}}{1 - K_{V_TL}G_D}G_{yL} \tag{35}$$

this transfer function exhibits a *gain/dynamic competing behavior between G_{yV} and G_{yL}*. In other words, this transfer function actually consists of the sum of two different transfer functions which have different signs in gain and different dynamics and, thus, can be shown to exhibit an inverse response. To illustrate this feature, if we take a very simple case with perfect level controllers ($G_D = G_B = 1$), constant molar overflow ($K_{V_T}L = 0$, $K_{V_TV} = 1$), and no time delay, and assume

$$G_{yL} = \frac{K_{yL}}{\tau_{yL}s + 1} \quad G_{yV} = \frac{K_{yV}}{\tau_{yV}s + 1} \tag{36}$$

then the numerator of G_{yV}^{DV} and the location of the single zero are

$$N(s) = s + \frac{K_{yL} + K_{yV}}{\tau_{yV}K_{yL} + \tau_{yL}K_{yV}}$$

$$s = -\frac{(K_{yL} + K_{yV})}{(\tau_{yV}K_{yL} + \tau_{yL}K_{yV})} \tag{37}$$

Thus the transfer function can have a zero in the right half plane (RHP) and y will exhibit an inverse response to changes in V if $(K_{yL} + K_{yV})$ and $(\tau_{yV}K_{yL} + \tau_{yL}K_{yV})$ have different signs. However, this situation rarely occurs unless the two time constants are very different from each other.

If imperfect inventory control (proportional-only control) and non-constant molar overflow conditions are considered in addition to the previous simple case, Eq. 35 becomes

$$G_{yV}^{DV} = \frac{K_{yV}}{\tau_{yV}s + 1} + \frac{K_{V_TV}}{(\tau_D s + 1 - K_{V_TL})}\frac{K_{yL}}{(\tau_{yL}s + 1)} \tag{38}$$

where τ_D is the time constant of transfer function, G_D, as derived in appendix A. By rearranging Eq. 38, the following characteristic equation for the numerator of G_{yV}^{DV} can be obtained:

$$N(s) = s^2 + \left(\frac{1}{\tau_{yL}} + \frac{1 - K_{V_TL}}{\tau_D} + \frac{K_{V_TV}K_{yL}\tau_{yV}}{K_{yV}\tau_{yL}\tau_D}\right)s + \frac{K_{yV}(1 - K_{V_TL}) + K_{yL}K_{V_TV}}{K_{yV}\tau_{yL}\tau_D} \tag{39}$$

If an odd number of zeros of the transfer function are in the RHP, then the transfer function exhibits an inverse response. For Eq. 39, if $(K_{yL}K_{V_TV}/K_{yV}$ is smaller than $K_{V_TL} - 1$, one root is in the RHP and the other is in the LHP regardless of the other parameter values because the leading coefficient of Eq. 39 is positive and the time constants are positive. Thus the transfer function (Eq. 38) has one RHP zero and y will exhibit an inverse response to changes in V under the conditions mentioned above.

These two simple cases involve simplifying assumptions, but both demonstrate that an inverse response can occur in the top product composition with a DV control structure under certain conditions. With further investigation, inverse response can be shown to occur in the bottom composition of the DV structure for a step change in boilup, and also in both top and bottom compositions for feed disturbances depending on the process characteristics. Also, inverse responses can occur with the LB structure. These results have been verified in three different ways for the UCSB column: with a transfer function model, with a rigorous dynamic simulation (a physically-based model), and through experiments. These results will be shown in the next section.

It is well known that distillation columns can exhibit an inverse response in bottom composition and bottom level dynamics for changes of boilup. Also Buckley et al. (1985), Shinskey (1984), and Waller et al. (1988b) discuss the inverse responses due to control loop interactions for some control structures under the special condition that one composition control loop is closed and the other is open. But these are not true "open-loop" inverse responses; thus the present paper seems to report for the first time that open-loop inverse responses due solely to the inventory control structure can be predicted and demonstrated.

Table 2. Steady-State Operating Conditions for the UCSB Simulation Model

Steady-State Conditions in mol/min

$F = 11.2$ $D = 3.01$ $B = 8.19$ $L = 13.0$ $V = 19.8$

Reflux ratio = 4.32 Reboiler heat duty = 47,500 kJ/h

	Compositions, mol %		
	n-Butanol	sec-Butanol	tert-Butanol
Feed	38.6	35.5	25.9
Distillate	0.234	17.9	81.9
Bottom	52.7	42.0	5.33

This particular result is just one indication of the capability and utility of the new structural transformation for dynamic models.

Description of Experimental Process and Simulation Models

A pilot-scale 12-stage sieve tray distillation column located at UCSB was used to verify the theory developed above. Both the experimental column and a dynamic simulator emulating operations of this column were used. The column is 15 cm in diameter and is equipped with a total condenser and a thermosiphon reboiler to separate a ternary mixture of *n*-butanol, *s*-butanol and *t*-butanol. The location of the feed stage is the fourth stage from the bottom tray. The compositions of the products are analyzed by an on-line gas chromatograph. Nominal operating conditions are given in Table 2.

The dynamic simulator is a full-order rigorous model with tray-by-tray calculations written in FORTRAN. It uses both an explicit Runge-Kutta method and a second-order semi-implicit Runge-Kutta method in the integration algorithm. Modeling assumptions include perfect mixing in each tray, negligible vapor holdups on each tray, and constant column pressure. For the vapor-liquid equilibrium calculations, the SRK equation and the Wilson equation are used for the vapor and liquid phase, respectively.

A transfer function model of the LV structure was obtained by open-loop tests of the dynamic simulator, fitting first-order plus time-delay models. (Any potential time delay caused by composition analysis is neglected.) The impurities, which are

Table 3. Transfer Functions of the LV Structure for the UCSB Simulation Model for Impurity Control*

$$\begin{bmatrix}\Delta y\\ \Delta x\end{bmatrix} = \begin{bmatrix}\dfrac{-3.58e^{-2.1s}}{19.2s+1} & \dfrac{3.38e^{-2.2s}}{19.8s+1}\\[2mm] \dfrac{13.9e^{-1.2s}}{13.2s+1} & \dfrac{-18.9e^{-0.6s}}{12.6s+1}\end{bmatrix}\begin{bmatrix}\Delta L\\ \Delta V\end{bmatrix} + \begin{bmatrix}\dfrac{-0.515e^{-3.7s}}{27.6s+1} & \dfrac{3.05e^{-4.9s}}{22.8s+1}\\[2mm] \dfrac{9.97e^{-0.4s}}{10.8s+1} & \dfrac{-28.1e^{-1.5s}}{14.5s+1}\end{bmatrix}\begin{bmatrix}\Delta F\\ \Delta z\end{bmatrix}$$

$$\begin{bmatrix}\Delta D\\ \Delta B\end{bmatrix} = \begin{bmatrix}\dfrac{-0.775}{1.14s+1} & \dfrac{0.818}{1.14s+1}\\[2mm] \dfrac{0.775}{1.75s+1} & \dfrac{-0.818}{1.75s+1}\end{bmatrix}\begin{bmatrix}\Delta L\\ \Delta V\end{bmatrix} + \begin{bmatrix}\dfrac{0.0382}{1.14s+1} & \dfrac{-1.20}{1.14s+1}\\[2mm] \dfrac{0.962}{1.75s+1} & \dfrac{1.20}{1.75s+1}\end{bmatrix}\begin{bmatrix}\Delta F\\ \Delta z\end{bmatrix}$$

*Obtained by fitting step test data from the dynamic simulation.
P-only controllers were used for both top and bottom level control.
($K_{cD} = -2.48$ mol/min/cm; $K_{cB} = -2.98$ mol/min/cm; $A_D = 283.4\ cm^2$; $A_B = 542.6\ cm^2$)
The time constants and time delays are expressed in minutes; all composition gains have been multiplied by 10^3.

Table 4. Transfer Functions of the DV Structure for the UCSB Simulation Model for Impurity Control*

$$\begin{bmatrix}\Delta y\\ \Delta x\end{bmatrix} = \begin{bmatrix}\dfrac{-4.63e^{-2.1s}}{(19.2s+1)(1.47s+1)} & \dfrac{3.38e^{-2.2s}}{19.8s+1} + \dfrac{-3.78e^{-2.1s}}{(19.2s+1)(1.47s+1)}\\[2mm] \dfrac{-18.0e^{-1.2s}}{(13.2s+1)(1.47s+1)} & \dfrac{-18.9e^{-0.6s}}{12.6s+1} + \dfrac{14.7e^{-1.2s}}{(13.2s+1)(1.47s+1)}\end{bmatrix}\begin{bmatrix}\Delta D\\ \Delta V\end{bmatrix}$$

$$+ \begin{bmatrix}\dfrac{-0.515e^{-3.7s}}{27.6s+1} + \dfrac{-0.712e^{-2.1s}}{(19.2s+1)(1.47s+1)} & \dfrac{3.05e^{-4.9s}}{22.8s+1} + \dfrac{0.727e^{-2.1s}}{(19.2s+1)(1.47s+1)}\\[2mm] \dfrac{7.97e^{-0.4s}}{10.8s+1} + \dfrac{0.687e^{-1.2s}}{(13.2s+1)(1.47s+1)} & \dfrac{-28.1e^{-1.5s}}{14.5s+1} + \dfrac{-2.86e^{-1.2s}}{(13.2s+1)(1.47s+1)}\end{bmatrix}\begin{bmatrix}\Delta F\\ \Delta z\end{bmatrix}$$

$$\begin{bmatrix}\Delta L\\ \Delta B\end{bmatrix} = \begin{bmatrix}\dfrac{-1.29}{1.47s+1} & \dfrac{1.06}{1.47s+1}\\[2mm] \dfrac{-1}{(1.47s+1)(1.75s+1)} & \dfrac{-1.21s}{(1.47s+1)(1.75s+1)}\end{bmatrix}\begin{bmatrix}\Delta D\\ \Delta V\end{bmatrix} + \begin{bmatrix}\dfrac{0.0493}{1.47s+1} & \dfrac{-1.55}{1.47s+1}\\[2mm] \dfrac{1.42s+1}{(1.47s+1)(1.75s+1)} & \dfrac{1.77s}{(1.47s+1)(1.75s+1)}\end{bmatrix}\begin{bmatrix}\Delta F\\ \Delta z\end{bmatrix}$$

*Obtained by transformation of the LV model in Table 3.

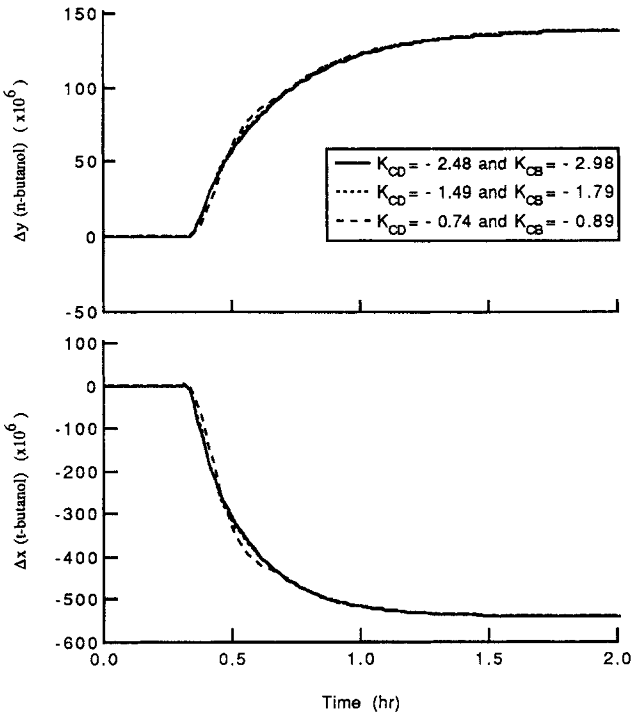

Figure 3. Open-loop responses of DV structure for a step change of $\Delta D = 0.0301$ mol/min with different PI-level controller settings using the transfer function model ($\tau_{ID} = 2.52$ min, $\tau_{IB} = 3.90$ min).

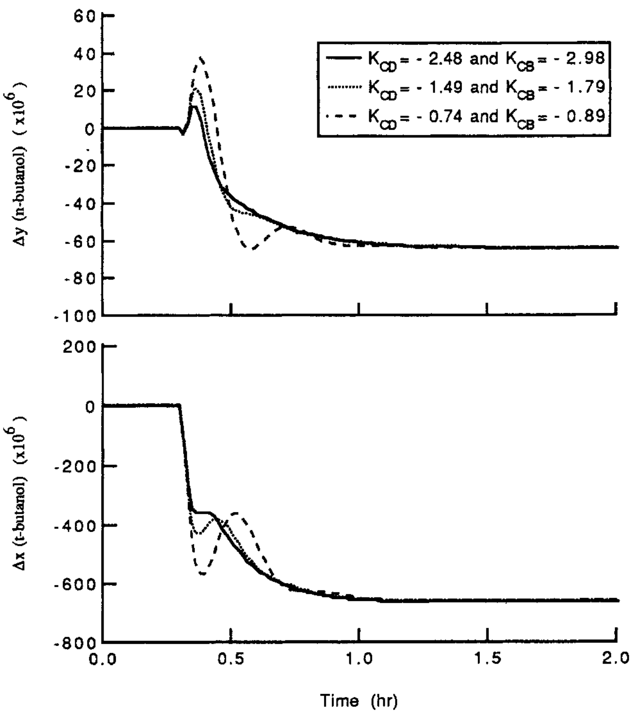

Figure 4. Open-loop responses of DV structure for a step change of $\Delta V = 0.160$ mol/min with different PI-level controller settings using the transfer function model ($\tau_{ID} = 2.52$ min, $\tau_{IB} = 3.90$ min).

normal butanol for the top product and tertiary butanol for the bottom product, are controlled by the manipulated variables (reflux rate and boilup rate); feed flow rate and feed composition of normal butanol are considered as disturbances. The transfer function model obtained for the LV structure from simulation results is shown in Table 3 except that the proportional-only level controllers are replaced with proportional-integral controllers. A transfer function model for the DV structure was obtained by the transformation technique of this paper and is shown in Table 4. Both of these transfer function models were used in subsequent evaluations discussed below.

Simulation and Experimental Results

A number of important features now can be demonstrated via simulated and experimental step changes. In each case, step changes were introduced after 0.3 h.

Importance of the Inventory Control Effects for Certain Control Structures. Figures 3 and 4 compare three inventory controller settings for the DV structure using the transfer function model. The LV structure is insensitive to inventory control as indicated earlier, but the DV structure is quite sensitive to inventory control, especially for the change. These figures indicate that inventory controls should not be neglected in analyzing the dynamics of control structures, and also that inverse responses can result from these effects.

Validity of the Steady-State Material Balance around the

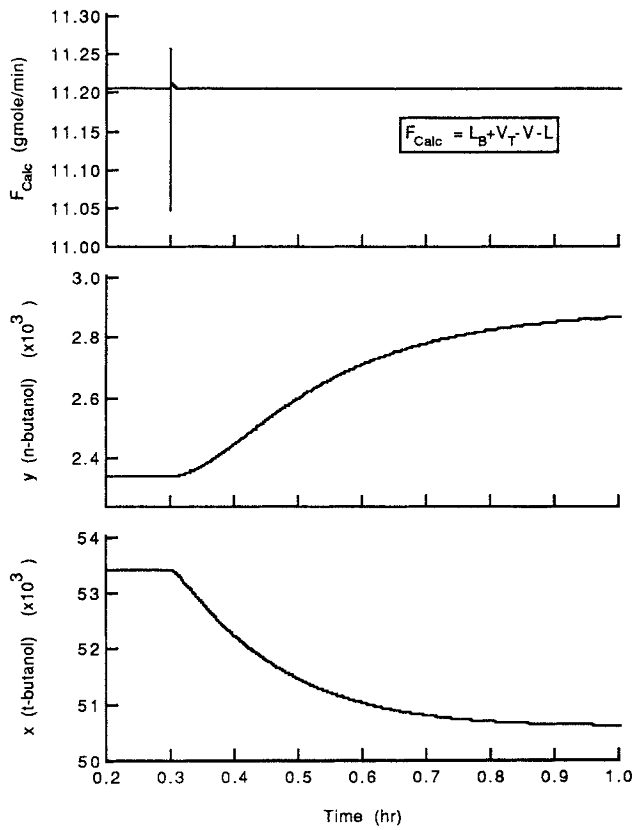

Figure 5. Dynamics of the material balance closure vs. composition dynamics for a step change of $\Delta V = 0.160$ mol/min using the LV structure with the rigorous dynamic simulator.

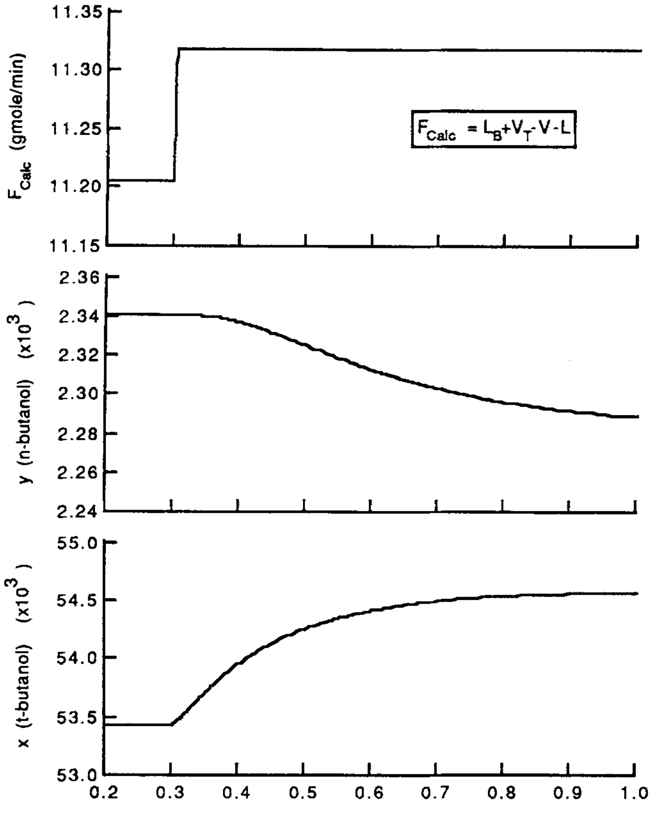

Figure 6. Dynamics of the material balance closure vs. composition dynamics for a step change of $\Delta F = 0.112$ mol/min using the LV structure with the rigorous dynamic simulator.

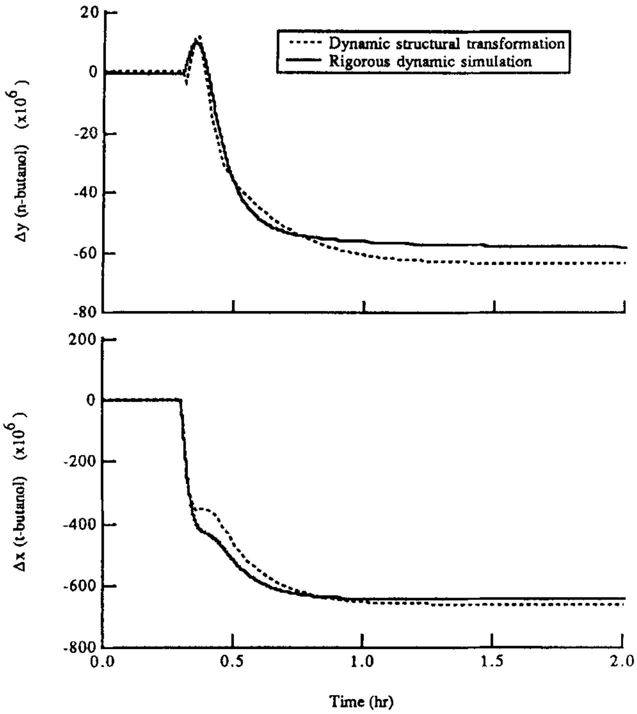

Figure 7. Open-loop responses for a step change of $\Delta V = 0.160$ mol/min of the DV structure using the rigorous dynamic simulation and the model obtained by dynamic structural transformation ($K_{CD} = -2.48$, $K_{CB} = -2.98$, $\tau_{ID} = 2.52$ min, $\tau_{IB} = 3.90$ min).

Invariant Element during Transient Responses. Results from the rigorous dynamic simulation described above indicate that further simplification of the internal flow dynamics can be achieved. The transient material balance around the invariant structural element is so fast compared to the composition dynamics that it can be neglected, as shown in Figures 5 and 6 for step changes in a manipulated variable (boilup) and a disturbance (feed flow rate). Thus the steady-state material balance can be assumed to be valid for the dynamic state, and then from the dynamic effects of manipulated or disturbance variables on ΔV_T (or ΔL_B), the dynamic effects on ΔL_B (or ΔV_T) can be calculated. For example, if the transfer function between ΔL_B and ΔL can be obtained from simulation or experiment, then the transfer function between ΔV_T and ΔL can be obtained directly from the material balance with little loss in accuracy. Since it is not easy to obtain all of the internal flow dynamic relations experimentally, this result can save time and effort in obtaining a better model.

Dynamic Transformation Technique vs. Full-Order Dynamic Simulator. Comparisons of responses using the rigorous nonlinear dynamic simulator and the transformed transfer functions for the DV structure are shown in Figures 7 and 8. The transformed transfer function model quite accurately predicts the detailed dynamic behavior of the full-order rigorous dynamic simulation for two different sets of level controller parameters. The differences in final steady states in Figures 7 and 8 are due to nonlinearities captured by the full-order dynamic simulator which are not incorporated in the linear transfer function model.

Inverse Responses due to the Control Structure. As indicated in Figures 4, 7 and 8, both the transfer function model and the full-order dynamic simulation of the DV structure exhibit an inverse response in top composition resulting from a change of boilup rate. Figure 9 shows simulated inverse responses in the bottom composition for the LB structure caused by a change in reflux flow rate. Figure 10 provides experimental confirmation for the DV structure. These inverse responses have been reproduced in repeated experiments.

Conclusions

A simplified dynamic model for distillation columns based on practical considerations has been formulated and a dynamic transformation technique has been developed that incorporates composition and flow dynamics and a general inventory control scheme. The model is derived starting from an invariant structural element and then adding inventory holdup and controller dynamics. The importance of inventory controls on dynamic responses for any structure other than the LV structure is illustrated. For modeling simplicity, we can assume that the steady-state material balance around the invariant structural element is valid, not only at the steady state but also approximately in the transient state. This assumption is verified by rigorous dynamic simulation. The structural transformation technique provides a physical interpretation of the gains of the

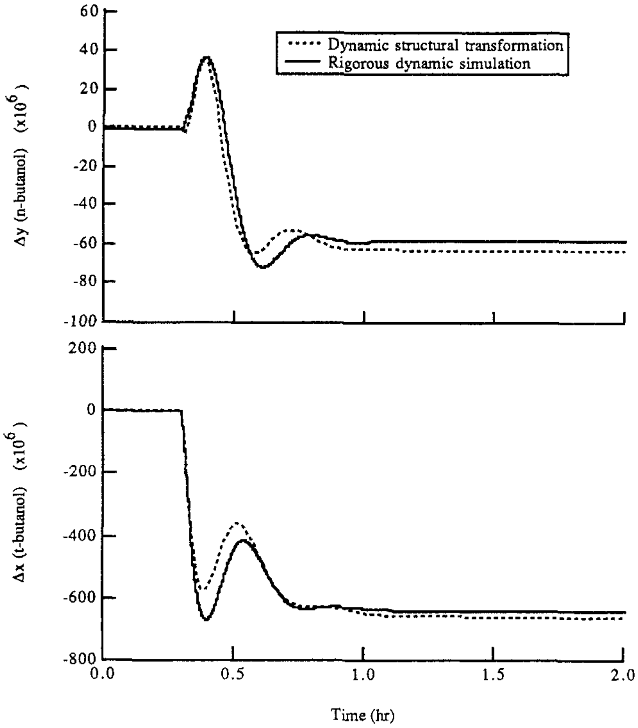

Figure 8. Open-loop responses for a step change of $\Delta V = 0.160$ mol/min of the DV structure using both the rigorous dynamic simulation and the model obtained by dynamic structural transformation ($K_{CD} = -0.74$, $K_{CB} = -0.89$, $\tau_{ID} = 2.52$ min, $\tau_{IB} = 3.90$ min).

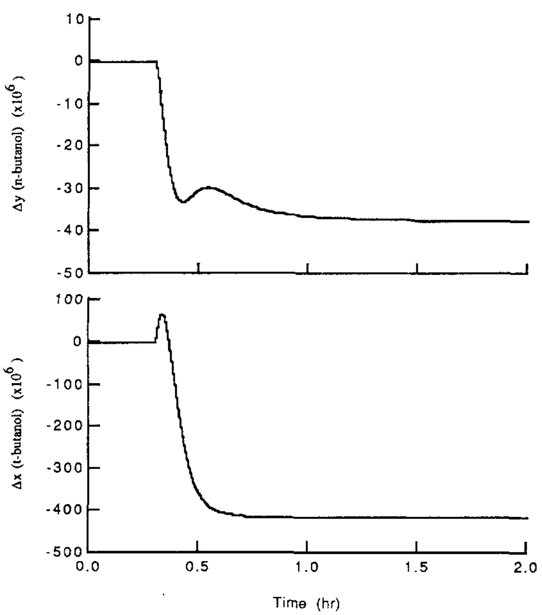

Figure 9. Simulation result for inverse response of LB structure for a step change of $\Delta L = 0.130$ mol/min using rigorous dynamic distillation column.

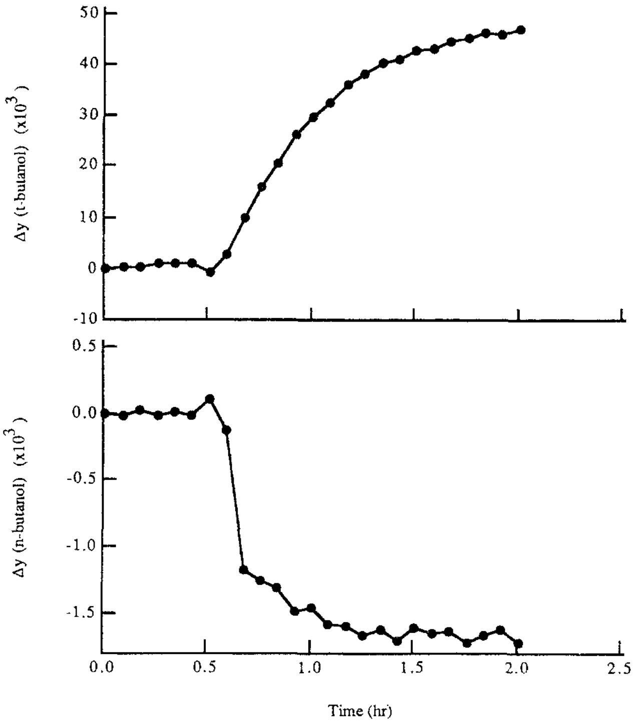

Figure 10. Experimental inverse response of DV structure for a step change in reboiler steam pressure from 75.9 to 93.1 kPa (11 to 13.5 psig).

internal flow dynamic relations, and a method to calculate these gains for arbitrary choices of the controlled components and disturbances for a multicomponent system. Thus a model for a base control structure (such as the LV structure) can be developed by obtaining the step responses for changes in manipulated variables and disturbances. Once the model for the base structure is developed, then without performing any additional experiments, process models for alternative control structures can be obtained easily by a dynamic structural transformation.

This technique's predictive capabilities have been verified experimentally for several interesting situations involving inverse dynamic responses that result from competing gain/dynamic effects. This research indicates that inverse response characteristics can be observed in product compositions for particular control structures. For example, with the DV structure, an inverse response in top composition can be observed for changes in boilup, feed rate, or feed composition. Also, with the LB structure, an inverse response in bottom composition can result for changes in reflux, feed rate, or feed composition. These inverse responses were predicted by theoretical analysis of a simplified model and verified by use of a rigorous (full-order) dynamic model. For the DV structure, the inverse response in top composition caused by a change in boilup also has been verified experimentally using the UCSB pilot-scale distillation column.

Acknowledgment

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Notation

A = cross-sectional area of reflux drum or reboiler
B = bottom product flow rate
D = distillate flow rate
e = error signal for the controller in block diagram
F = feed flow rate
$\mathbf{G}$ = transfer function matrix
G = transfer function
$\mathbf{H}$ = coefficient matrix for structural transformation
h = liquid level in reflux drum
$\mathbf{I}$ = inventory control manipulators for drum level and reboiler level
i = index for a component
j = index for a component
K = gain
k = index for a disturbance component
L = reflux flow rate
L_B = liquid flow rate from the bottom stage to reboiler
N = numerator polynomial
n = total number of components
p = controller output in block diagram
s = Laplace transform variable
$\mathbf{U}$ = vector of composition manipulators
$\mathbf{V}$ = vector of inventory manipulators
V = boilup rate from the reboiler
V_T = vapor flow rate from the top stage to condenser
$\mathbf{W}$ = vector of disturbance variables
x = bottom product composition
$\mathbf{Y}$ = vector of controlled composition variables
y = distillate composition
z = feed composition

Greek letters

Δ = deviation from a nominal steady-state value
μ = vector of new composition manipulators
ν = vector of new inventory manipulators
ρ_M = density in reflux drum or reboiler
τ = time constant
ω = vector of new disturbance variables

Subscripts

B = bottom product flow
D = distillate flow
L = reflux flow
n = normal butanol
T = vapor from the top stage
t = tertiary butanol
V = boilup
x = bottom product composition
y = distillate composition

Superscripts

DV = DV structure
T = transpose
o = invariant structure
$*$ = modified transfer function corresponding to the new inventory control structure

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Appendix A: Derivation of G_D and G_B for Proportional and Proportional-Integral Control

In Figure 2, the mass process function of the reflux drum, G_P, is obtained from the following mass balance,

$$\rho_{MD} A_D \frac{dh}{dt} = \Delta V_T - \Delta L - \Delta D \tag{A1}$$

where ρ_{MD} is the molar density of the distillate, A_D is the cross-sectional area of the reflux drum, and h is the drum level.

Using Laplace transforms, Eq. A1 becomes

$$G_P(s) = \frac{h(s)}{\Delta V_T(s) - \Delta L(s) - \Delta D(s)} = \frac{1}{\rho_{MD} A_D s} \quad \text{(A2)}$$

For simplicity, assume $G_m(s) = G_V(s) = 1$ and ρ_{MD} is constant. If a level controller is used, then

$$\Delta p = \Delta D = G_C \Delta e \quad \text{(A3)}$$

where Δp is the controller output, Δe is the output error from the setpoint comparator, and G_C (the controller transfer function) is equal to K_{CD} for proportional-only control.

In the LV structure, ΔD is used to control the level of the reflux drum, so the transfer function (G_D) between ΔD and $\Delta V_T - \Delta L$ can be derived. For constant setpoint, $\Delta e = -\Delta h$ and the following relation is obtained from Eqs. A2 and A3,

$$G_D = \frac{\Delta D}{\Delta V_T - \Delta L} = \frac{1}{\tau_D s + 1} \quad \text{(A4)}$$

where $\tau_D = \rho_{MD} A_D/(-K_{CD})$ and K_{CD} is negative. If a PI controller is used, then

$$G_C = K_{CD}\left(1 + \frac{1}{\tau_{ID} s}\right) \quad \text{(A5)}$$

For constant setpoint, the relation between ΔD and $\Delta V_T - \Delta L$ is obtained from Eqs. A2, A3 and A5:

$$G_D = \frac{\Delta D}{\Delta V_T - \Delta L} = \frac{\tau_{ID} s + 1}{\dfrac{\tau_{ID} \rho_{MD} A_D}{(-K_{CD})} s^2 + \tau_{ID} s + 1} \quad \text{(A6)}$$

In similar fashion, G_B can also be derived by replacing ρ_{MD}, A_D, K_{CD}, and τ_{ID} with ρ_{MB}, A_B, K_{CB}, and τ_{IB}, respectively.

Appendix B: Derivation of $K_{V_T L}$, $K_{V_t V}$, $K_{V_T F}$, and $K_{V_T z}$

Constants, $K_{V_T L}$, $K_{V_T V}$, $K_{V_T F}$, $K_{V_T z}$ are obtained from steady-state relationships. From Eqs. 3, 12 and 16, if only ΔL is considered in order to calculate $K_{V_T L}$ with $\Delta V = \Delta F = \Delta z = 0$, then

$$\Delta y_i = K_{y_i L} \Delta L \quad (i = 1, \ldots, n) \quad \text{(B1)}$$

$$\Delta x_i = K_{x_i L} \Delta L \quad (i = 1, \ldots, n) \quad \text{(B2)}$$

$$\overline{y}_i \Delta D + \overline{D} \Delta y_i + \overline{B} \Delta x_i + \overline{x}_i \Delta B = 0 \quad (i = 1, \ldots, n) \quad \text{(B3)}$$

$$\Delta D = (K_{V_T L} - 1) \Delta L \quad \text{(B4)}$$

$$\Delta D = -\Delta B \quad \text{(B5)}$$

From Eqs. B1 – B5, $K_{V_T L}$ in Eq. 17 is obtained:

$$K_{V_T L} = 1 - \left(\frac{\overline{D} K_{y_i L} + \overline{B} K_{x_i L}}{\overline{y}_i - \overline{x}_i}\right) \quad (i = 1, \ldots, n) \quad (17)$$

In the same way, $K_{V_T V}$, $K_{V_T F}$, and $K_{V_T z}$ in Eqs. 18–20 can be determined.

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